The concept of a process is technological in nature. Depending upon available technologies, several processes are available to a firm for producting a particular product. Each production process can be used at several levels to produce a particular product. The higher the level at which a process is used, the greater the output produced. It is worth mentioning that one process can be substituted for another. Besides, when a commodity is produced with two or more processes at the same time in a community, it is generally assumed that the two processes do not interfere with each other or do not increase the productivity of one another. A typical linear programming problem is to choose a process or a combination of processes that, given the constraints, minimizes cost of production.

Representation of Processes : Process Rays

Given the linearity assumption, in each production process proportion between factors remains fixed or constant. As a result, each production process can be expressed through a straight line passing through the origin. In a diagram whose two axes represent the two factors, say labour and capital, the straight line passing through the origin which is the locus of points involving fixed proportions between the two factors is called a *process ray*. In Fig. 17.1 a process ray OA has been drawn which represents a fixed ratio between capital and labour which is given by the slope of the line. It may be noted that on the *process ray OA* points Q_1, Q_2, Q_3 etc., are the levels at which the given process can be used. It will be further noticed that the process A, is a relatively capital-intensive method of production. Similarly, in Fig. 17.1 process rays OB, OC and OD representing different factor proportions and therefore representing different processes B, C and D have been drawn which are respectively less and less capital intensive. R_1, R_2, R_3 etc., are the different levels of the process B, S_1, S_2, S_3 etc. are the different levels of process C, and T_1, T_2, T_3 etc., are the different levels of process D. Because linear relationship between inputs and outputs has been assumed, as the amount of inputs or factors are increased along a given process ray, output will increase in the same proportion as inputs, that is, constant returns to scale will occur. Since points Q_2, Q_3 and Q_4 have been taken at the same distance from each other, the increment in inputs between them will be the same.

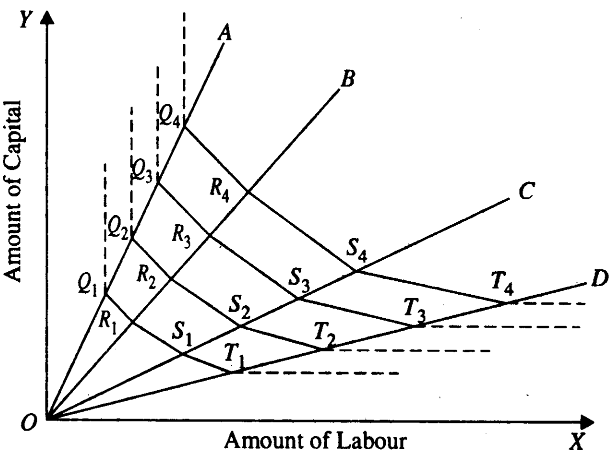


Fig. 17.1. *Process Rays*

On process ray OB, points R_1, R_2, R_3 and R_4 lying on it represent a different factor proportion (*i.e.* capital-labour ratio) from that on various points at process ray OA. It is worth mentioning that it is not necessary that to produce a given output along process ray B, one has to work at the same level of process B as of A, that is, distance OR_1 on OB may not be equal to the distance OQ_1 on OA; both Q_1 and R_1 yielding the same level of output. Likewise, factor combination R_2 and Q_2 which yield the same level of output may not be equidistant from the origin O.

Similarly, points S_1, S_2, and S have been taken on the process ray OC and points T_1, T_2, T_3 and T_4 have been taken on process ray OD. With factor combination S_1 of process C, T_1 of process D, output equal to that of Q_1 or R_1 is obtained. With factor combination S_2 of process C and factor combination T_2 of process, output equal to that of Q_2 or R_2 is obtained.

Likewise, on process ray OD, points T_1, T_2, T_3, T_4 represents output equal to Q_1, Q_2, Q_3 and Q_4 respectively. On joining points Q_1, R_1, S_1 and T_1, we get a kinked curve $Q_1R_1S_1T_1$ (with linear segments) which is called an *isoquant* and is similar to equal product curves of the traditional production theory. Further, points Q_2, R_2, S_2 and T_2 are joined together to obtain another isoquant $Q_2R_2S_2T_2$ representing a higher level of output. Likewise, higher isoquants are obtained by joining points on different process rays. In order to make it more similar to the iso-product curves of traditional production theory, the extremes of isoquants of linear programming theory are usually extended by vertical and horizontal lines as shown in Fig. 17.1 by the dotted lines. Another point worth mentioning is that iso-product curves or isoquants of traditional production theory are smooth or continuous, whereas isoquants in linear programming theory are kinked having right-line segments.

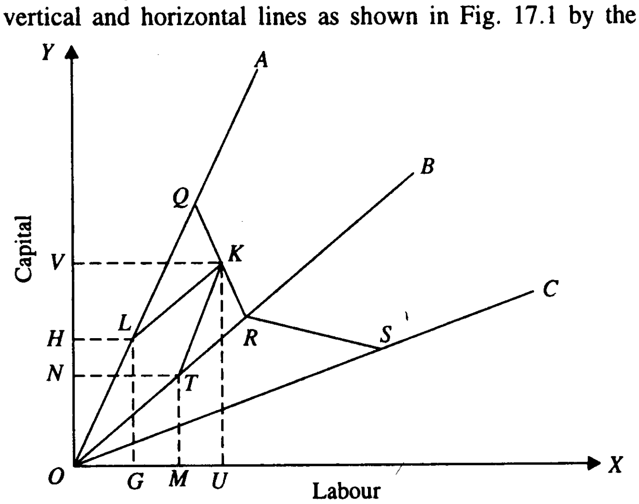


Fig. 17.2. *Combination of Processes*

Combination of Processes

As mentioned above, more than one processes are generally available to produce a commodity and various levels of output of the commodity can be produced by working at different levels of the processes. It is worthwhile to note that instead of producing a commodity with a single production process a combination of two processes can also be used to produce a commodity. A part of the output of a commodity can be produced with one process and a part with another process. This happens when a firm is working at a point like K on segment QR in Fig. 17.2. In this figure three process rays, OA, OB and OC have been depicted and an isoquant QRS has been drawn. If a firm decides to work at point K to produce a given level at output represented by the isoquant QRS, it will employ two processes A nd B to produce the commodity. Now, the question is at what levels of the processes A and B the firm will work when it is producing at K. In order to know this a line from point K parallel to OB is drawn which meets OA at L and, similarly, a line from K parallel to OA is drawn which meets OB at T. Thus, to produce the output corresponding to point K on isoquant QRS, the firm will work at the level OL of process A and at the level OT of process B.

It can be proved that the total amounts of factors, labour and capital, used at points L and T will be equal to the factor combination represented by K (*i.e.*, OV of capital and OU of labour). That is, $OG + OM = OU$ of labour and $OH + ON = OV$ of capital.[1]

In the same way, working at any point on the straight-line segment between R and S will mean that a combination of two processes B and C will be used for the production of the commodity. However, it may be noted that working at the corner points such as Q, R or S means that only one process will be used for the production of the commodity; working at point Q involves the use of process A only, at point R involves the use of process of B only, and at point S involves the use of process C only.

Objective Function

Objective function, also called criterion function, describes the "determinants of the quantity to be maximised or to be iminimised."[2] If the objective of a firm is to maximise output or profits, then this is the objective function of the firm. If the linear programming requires the

1. For proof See J. Baumol, *Economic Theory and Operations Analysis*, 4th edition, p. 305.
2. D.S. Watson. *Price Theory and Its Uses*, 4th edition, p. 206.

minimisation of cost, then this is the objective function of the firm. An objective function has two parts—the primal and dual. If the primal of the objective function is to maximise output with a given cost, then its dual will be the minimisation of cost for producing a given output.

Constraints

The maximisation of the objective function is subject to certain limitations which are called constraints. The budget or income of a consumer is constraint on him for maximizing his satisfaction. A firm which aims to maximize its output is constrained by the fact that it has, say, only 13 machines to work with and a certain limited floor space on which work has to be performed. Besides, constraints on a firm may be of the type that for a particular machine say *A*, at least two labourers are needed to operate it, and for another machine *B*, at least 5 labourers are required to operate it. *Constraints are also called inequalities* because they are generally expressed in the form of inequalities. The constraint regarding the availability of 10 machines for production is generally expressed as 10 or fewer machines (that is $\leq$ 10 machines) are available for production. If the constraint is that at least two workers are required to operate a machine, then it is written as $\geq$ 2 workers are needed to operated a machine.

Feasible Solution

After knowing the constraints, feasible solutions of the problem for a consumer, a producer, a firm or an economy can be ascertained. *Feasible solutions are those which meet or satisfy the constraints of the problem and therefore it is possible to attain them.* For a consumer which aims to maximize his satisfaction from his purchases of goods, feasible solutions are those which lie at or to the left of the given price line, which in turn is determined by the constraints of the given income and the given prices of goods. Likewise, feasible solutions for a firm using two inputs, labour and capital, and seeking to maximize his output, feasible solutions are those possible combinations of inputs which lie on or to the left of the given iso-cost line which in turn is determined by the given total outlay and given prices of the two inputs. The shape of the area or region of feasible solutions depends upon the nature of constraints. If the constraints of a firm for the production of a commodity are given by a certain amount of physical capital, say, *OM*, and a certain amount of labour, say, *OL*, then the region of feasible solutions is represented in Fig. 17.3 by the area *OMKL*. Since no more than *OL* amount of labour and no more than *OM* amount of capital is available in Fig. 17.3 a vertical line *LL′* at *OL* amount of labour has been drawn to represent the constraint of labour and a horizontal straight line *MM′* at the amount of capital *OM* has been drawn to represent the constraint of capital. Any combination of factors which lies on or within the rectangle *OMKL* is feasible to be used for production. And any combination of inputs which is outside the region *OMKL* is not feasible to be used for production.

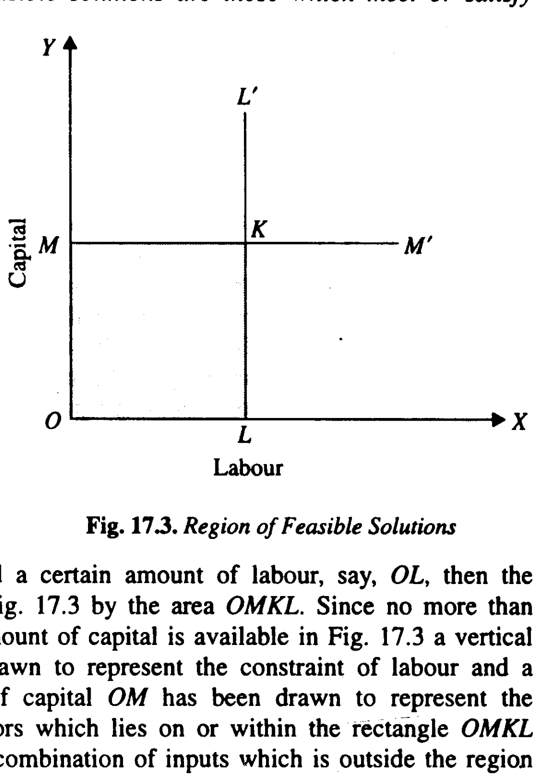


Fig. 17.3. *Region of Feasible Solutions*

Another example of the region of feasible solutions is depicted in Fig. 17.4. It has been assumed that two goods *X* and *Y* are to be produced. A machine *A* which is used for the production of a commodity has the capacity to work for 12 hours a day and the machine *B* has the capacity to work for eight hours a day. The production of a unit of commodity *X* requires 2 hours work each on machine *A* and *B*. To produce a unit of commodity *Y* requires three hours work on machine *A* and one hour work on machine *B*. These constraints can be expressed in the form of following inequalities:

$$2X + 3Y \leq 12$$

$$2X + 1Y \leq 8$$

These constraints are represented in Fig. 17.4. If the entire capacity of 12 machine hours of machine *A* is devoted to the production of *X*, then 6 units of commodity *X* are produced. If entire capacity of 12 hours of machine *A* are used for the production of commodity *Y*, then 4 units of product *Y* are produced. Thus, the line *AB* in Fig. 17.4 represents the constraint imposed by machine *A*, that is, it represents $2X + 3Y \leq 12$. The area to the left of the line *AB* represents the feasible region from the viewpoint of the given capacity of machine *A*.

In the same way, constraint $2X + 1Y \leq 8$ regarding the available capacity of machine *B* has been represented in Fig. 17.4 by the line *CD*, the area to the left of which represents the area of feasible solutions from the viewpoint of the capacity of machine *B*. But from the viewpoint of the constraints imposed by the two machines together, the region of feasible solutions is given by the shaded area *OAKD*. We have given above some examples of the region of feasible solutions. The shape of the feasible region would differ as the number and nature of constraints vary.

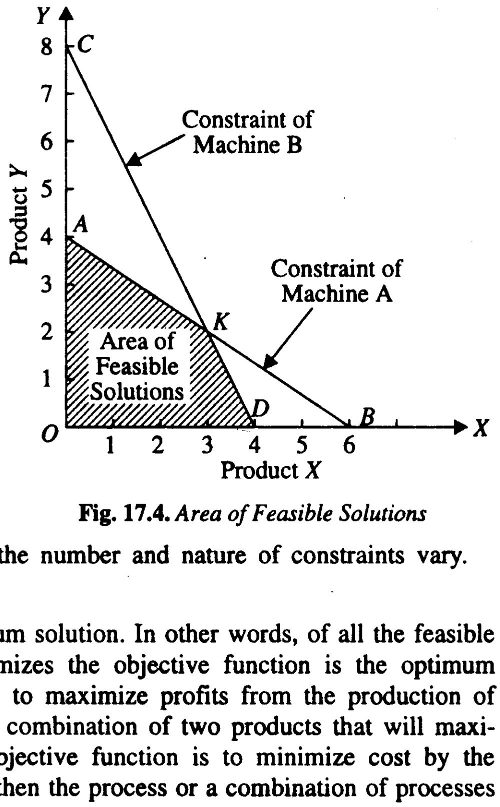


Fig. 17.4. *Area of Feasible Solutions*

Optimum Solution

The best of all feasible solutions is the optimum solution. In other words, of all the feasible solutions, the solution which maximizes or minimizes the objective function is the optimum solution. For instance, if the objective function is to maximize profits from the production of two goods, then the optimum solution will be the combination of two products that will maximize the profits for the firm. Similarly, if the objective function is to minimize cost by the choice of a process or a combination of processes, then the process or a combination of processes which actually minimizes the cost will represent the optimum solution. It is worthwhile to repeat that optimum solution must lie within the region of feasible solutions.

In linear programming there are two alternative methods of finding the optimum solution. One is the non-mathematical or *graphical method* of obtaining the optimal solution. This graphical method can handle only simple linear programming problems. The other method of finding the optimum solution of the linear programming problem is the *simplex method.* The simplex method involves a set of successive marginal calculations with which feasible solutions are successively tested during which the poorer solutions (*i.e.* which are not optimal) are successively eliminated until ultimately the optimum solution is obtained. Thus, the simplex method uses a mathematical and computational procedure to find out the optimal solution for the problem in question. Though the graphical and simplex methods follow different procedures to obtain the optimal solution for the problem, they reach identical numerical results.

CHOICE OF PRODUCTS : MAXIMIZATION OF PROFITS

An important linear programming relates to the maximization of profits in the production of two products when it is subject to some constraints. That is, what quantities of the two products are produced so that the profits of the firm are maximized when their production is subject to some constraints. Such analysis is highly relevant for a multiple product firm *i.e.*, for a firm which produces more than one product. In this regard it is worthwile to know that production is subject to what type of constraints. Let us suppose that the production of two products X and Y require the use of two machines I and II. The available capacity of machine I is to work for 12 hours in a day and the capacity of machine II is to work for eight hours a day. The other constraint relates to the machine hours required for the production of each unit of commodities X and Y. Let us assume that to produce a unit of product X, 2 hours work each on machine I and II is required and to produce a unit of product Y, 3 hours work is required to be done on machine 1 and one hour work is needed to be done on machine II. We can write these two constraints in the following manner:

$$2X + 3Y \leq 12 \qquad ...(i)$$

$$2X + 1Y \leq 8 \qquad ...(ii)$$

The first constraint is shown by the straight line AB and the second constraint is shown by another straight line CD (How these straight lines representing the constraints are drawn has been explained above). The area to the left of the kinked line AQD represents the region of feasible solutions.

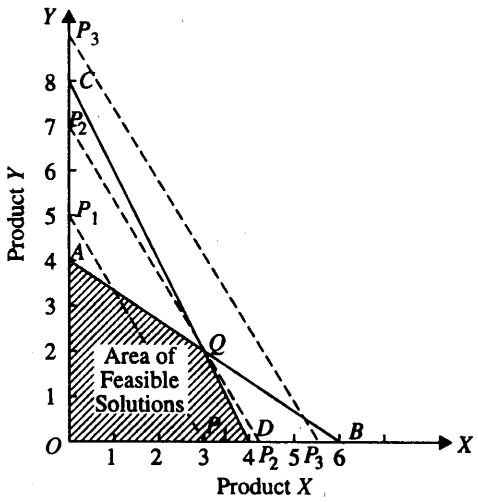


Fig. 17.5. *Choice of Products: Maximization of Profits*

Iso-Profit Curves. To show which combination of two products X and Y, the firm will produce so as to maximise its profits, it is necessary to explain first the concept of *iso-profit curves*. To draw iso-profit curves one needs to know the profit per unit produced the two products. *The profits or net revenue earned per unit of a product can be obtained by deducting the average variable cost from the price per unit of the product.* Let us assume that profits obtained from product X are Rs. 10 per unit of output and from product Y are Rs. 6 per unit of output. We can therefore write the objective functions as follows:

$$\pi = 10X + 6Y$$

where π (*i.e.*, profits) are to be maximised subject to the constraints stated above in (*i*) and (*ii*), X and Y represent the quantities of two products.

In order to represent the profits through iso-profit profit curves we shall have to fix the various amounts of profits. With different amounts of profits, the level of iso-profit curve will vary; the larger the amount of profits to be earned, the higher the level of iso-profit curve. If the amount of profits to be made is Rs. 30, then the equation of the objective function will be as follows:

$$30 = 10X + 6Y$$

If the product X is not to be produced at all and therefore its amount is zero, we will get the following equation of the objective function:

$$30 = 10(0) + 6Y$$

$$Y = 5$$

This means that with the production of 5 units of product Y and none of product X, amount of profits made will be Rs. 30. The point 5 is plotted on the Y-axis. Likewise, if no amount of product Y is produced, then

$$30 = 10X + 6(0)$$

$$X = 3$$

This means that the production of 3 units of X alone and none of Y will fetch the profit of Rs. 30. Therefore, point 3 is plotted on the X-axis. Thus, by joining the point 5 on the Y-axis with point 3 on the X-axis through a straight line, we get an iso-profit curve P_1P_1 representing the amount of profits equal to Rs. 30. All combinations of two products lying on the iso-profit curve P_1P_1 will yield profit equal to Rs. 30. Similarly, iso-profits curves P_2P_2, P_3P_3 showing successively higher levels of profits can be drawn. Given that profit per unit of products X and Y remain unchanged, various iso-profit curves will be parallel to each other.

Now, consider Fig. 17.5 wherein iso-profit curves have been superimposed on the region of feasible solutions. As noted above, the higher the level of iso-profit curve, the greater the amount of profits. Therefore, a firm whose objective is to maximise profits, will seek to go to the highest possible iso-profit curve. However, it cannot be beyond the region of feasible solutions because constraints prevent it to do so. A glance at Fig. 17.5 will reveal that the firm will make maximum possible profits by producing at point Q where an iso-profit curve P_2P_2 is touching the boundary of the region of feasible solutions. The firm will not produce at any other point since any point other than Q on or within the region of feasible solutions will lie at a lower iso-profit curve.

It is worthwhile to note that the optimum solution of the linear programming problem will always be at the boundary or corner of the region of feasible solutions. This follows from a simple logic even without reference to the iso-profit curves. It may be recalled that in linear programming constant returns to scale are assumed to be prevailing and also prices of products and inputs are assumed to remain constant. Therefore, if the production of a commodity is profitable, a firm will continue to expand output because neither diminishing returns to scale will occur nor there will be any adverse effects on the prices of output and inputs. Therefore, it will always be worthwhile to increase, output until *some capacity limit* is reached, that is, until the boundary of the feasible region is reached. Point Q is a corner point of the region of feasible solutions. The firm cannot go beyond or above the point Q because of the constraints. Hence, we conclude that by producing outputs of X and Y as represented by the point Q, the firm will make maximum profits. In other words, the production of combination of products of X and Y represented by point Q is an *optimum solution* for it. Point Q is a corner point of the feasible region. It is worth remembering that *when there is a single solution for the linear programming problem, it will always lie at the corner point.*

CHOICE OF A PROCESS : OUTPUT MAXIMIZATION SUBJECT TO SOME CONSTRAINTS

Another important problem of a firm which can be solved through the method of linear programming is that of choosing a best process for production of a commodity. In this analysis of choice of a process it is generally assumed that a firm seeks to maximize its output subject to the constraint (*i*) a given outlay or (*ii*) the given quantities of some available inputs. On the other hand, if the level of output is given, then choice of a process by a firm is analysed by assuming that a firm will aim to minimize cost for the given level of output. We shall discuss all these cases of the choice of a best process.

Choice of a Process : Output Maximization with Cost-Outlay Constant

Let us suppose that a firm uses two inputs, labour and capital to produce a commodity X. Further, four production processes are available to the firm for the production of the commodity X. These processes are represented by the process rays OA, OB, OC, and OD in Fig. 17.6 wherein along X-axis amount of labour is measured and along the Y-axis amount of capital is measured. The process rays, OA, OB, OC, and OD are successively more and more labour-intensive. Points L, M, N and W on the process rays OA, OB, OC and OD respectively represent the levels of these processes, the working at which yields the same level of output, say 40. By joining these points we get an isoquant $LMNW$ representing 40 units of commodity X. Likewise, we join points J, H, G, K, on various process rays to represent an isoquant representing output level of 60 units of commodity X.

Let us assume that with a given cost-outlay say T, the firm can buy OF amount of labour at its given price P_L or OE amount of capital at its given price P_K. Then, on a straight line EF, called an iso-cost line, any combination of two factors can be purchased with the given cost-outlay. Writing this in the form of equation:

$$T = L.P_L + K.P_K$$

where T stands for the given cost-outlay, L for the amount of labour, P_L for the given price of labour, K for the amount of capital and P_K for the given price of capital. With the constraint of a given total cost-outlay, the area to the left or on the line EF represents the feasible region. But, as noted above, since only four processes of production represented by process rays OA, OB, OC and OD are available, choice of process will be confined to the area within process rays OA and OD. Since with a given cost-outlay, a firm will seek to maximize production, the actual choice of process will be made from among the processes lying on the iso-cost line EF. It will be seen from the Fig. 17.6 that the iso-cost line EF is touching the corner point G of isoquant $JHGK$ representing 60 units of output. No point other than G lying on the given iso-cost line EF will yield greater output. Thus, the factor combination G is the optimum solution of the problem, that is, output-maximizing combination for the given cost-outlay. It will be observed that point G lies on the process ray OC. It follows therefore that firm will employ process C for the production of the commodity.

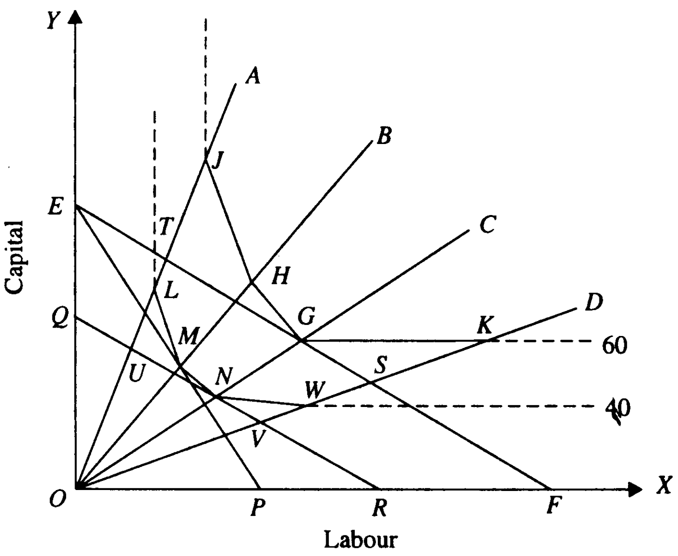


Fig. 17.6. *Choice of a Process*

Minimum Cost For a Given Level of Output : Dual Problem. It is worthwhile to mention here that if the problem is to choose a process subject to the given output of 60 units of product X, even then the firm will choose the point G on the process ray OC. In this case output of 60 units or isoquant $JHGK$ will be the constraint and the firm will aim at minimising cost for its production. It will be seen from Fig. 17.6 that on *the isoquant JHGK, point G will lie on the lowest possible iso-cost line EF*. It means cost-minimizing solution for the production of 60 units of output will also be G which, as seen above, is the output-maximizing solution for the cost-outlay represented by the iso-cost line EF. Since cost-minimization for producing a given output is the dual of the primal problem of output maximization with a given cost-outlay, it follows that the *optimum solution for the dual of the linear programming problem is the same as that of its primal problem.*

Effect of Change in Factor Price. Now a pertinent question is how the change in price of a factor (or input), the total cost-outlay and price of the other inputs remaining the same, will affect the choice of the process. Suppose in Fig. 17.6 as a result of the rise in the price of labour, total cost-outlay and price of capital remaining constant, the iso-cost line shifts to EP. It will be observed from the Fig. 17.6 that the new iso-cost line EP is touching the corner

M of the isoquant *LMNW* representing 40 units of output. Thus the input combination *M* is now the optimum solution. It will be seen that input combination *M* lies on the process ray *OB*. It means as a result of the rise in price of labour, the firm has shifted from the process *C* which is more labour-intensive to process *B* which is relatively less labour-intensive (or more capital-intensive). On the other hand, if the price of capital would have risen, price of labour remaining the same, the firm would have shifted to a more labour-intensive process than that represented by point *G*. Thus, the choice of an optimum process depends on the prices of inputs.

Effect of Changes in Outlay. Now, another interesting question is now the change in the total cost-outlay, prices of both inputs, capital and labour, remaining the same, will affect the choice of process. Suppose in Fig. 17.6, as a result of the fall in total cost-outlay iso-cost line shifts to a lower position *QR* which is parallel to the initial iso-cost-outlay *EF*. It will be seen from Fig. 17.6 that *QR* is touching the corner *N* of the isoquant *LMNW* and *N* lies on the same process line *OC* as point *G*. It means, *change in total cost-outlay, input prices remaining unchanged, will not affect the choice of the process; only the firm will now work at a different level of the same process.*

It is also clear from the aobve analysis that when there exists only one constraint, namely, total cost-outlay in the above case, there will be only one process which will be optimal and will be used by the firm for the production of a good.

Choice of a Process : Output Maximization with Two Inputs as Constraints

Now, if instead of cost-outlay as constraint, the firm is faced with constraints of two inputs which are available to it in limited quantities. Evidently, with the given amounts of inputs, the firm will try to maximise output so as to maximise its profits. Let us assume that three processes *A*, *B* and *C* are available for the production of the commodity produced by the firm. In Fig. 17.7 process rays *OA*, *OB* and *OC* represent the three processes *A*, *B* and *C*. *OL* and *OK* are the available quantities of labour and capital. Thus the horizontal straight line *KT* represents the constaint of available capital and the vertical line *LG* represents the constraint of available labour. These two lines interesect at point *S*. The area *OKSL* represents the region of feasible solutions.

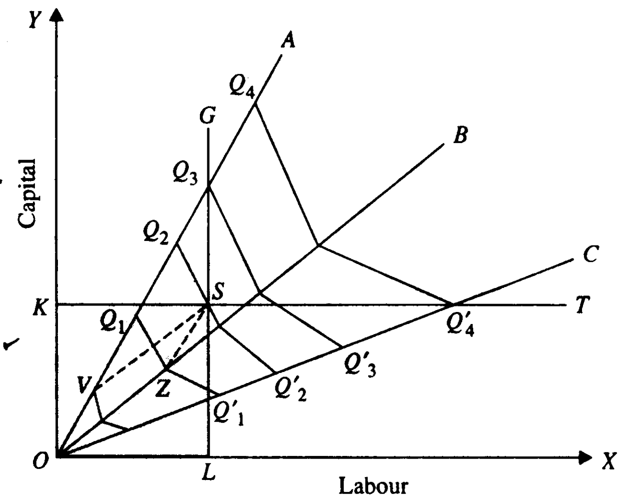


Fig. 17.7. *Output Maximisation with Two Inputs as Constraints*

As the firm will be interested in maximizing output with the given constraints, it will try to reach the highest attainable isoquant. So it will go to the boundary of the feasible region because production at this will ensure maximum output which is possible within the constraints. It will be observed from the Fig. 17.7 that the feasible region is touching the isoquant Q_2Q_2' at the corner point *S*. So the firm will produce maximum possible output represented by isoquant Q_2Q_2' by working at point *S*. The point *S* therefore is the optimum solution for the choice of the process. A glance at the Fig. 17.7 will reveal that the boundary or corner point *S* lies at the segment of the isoquant Q_2Q_2' in between the process rays *OA* and *OB*. It is thus clear that at the optimum solution point *S*, the firm will use a combination of two processes *A* and *B*. By drawing a line *SV* parallel to *OB* and line *SZ* parallel to *OA* we determine that the firm will work at the level *OV* of process *A* and at the level *OZ* of process *B* to reach the optimum input combination *S*.

Output-Maximisation with one Limited Input

Instead of two limited inputs, there may be one limited input which constitutes the constraint, while the other input is available in any desired amount. It may happen that, in case of capital, capacity of a machine is limited or a warehouse of a limited given capacity is available. Or, it may happen that a limited amount of labour is available, while there does not exist any constraint regarding capital. Consider Fig. 17.7 again. If labour is available in any amount desired by the firm but only *OK* amount of capital is available for production, the constraint line *LG* would not be there and only the horizontal line *KT* representing the constraint of limited capital will be there. A firm with its objective of maximization of output will move along the capital constraint line *KT* so as to reach highest achievable isoquant. Since only three processes are available, the firm cannot go to the right of process ray *OC*. It will be observed that with *OK* as limited capital the firm will go at the most upto point Q'_4 on process ray *OC*. Thus Q'_4 will be the optimum solution with only *OK* amount of capital as the constraint.

If there are no limitations in regard to the availability of capital, while labour was available only in limited quantity, such as the amount *OL* in Fig. 17.7, then *LG* will be the line representing the labour constraint. Now, the vertical line *LG* representing the given labour constraint meets the ray *OA* at point Q_3. Since only three process rays are available, the firm cannot go beyone Q_3 (*i.e.*, to the left) of process ray *OA*. Thus with *OL* as the maximum amount of labour available the firm will use the process *A* and work at its level OQ_3.

A Special Problem in Linear Programming : Diet Problem

In the modern days linear programming technique has been used for a wide types of business and social problems. One such social problem in which linear programming has been used to find and optimum solution is the problem of feeding diet to the animals. The problem of feeding optimum diet is the problem in minimizing cost of feeding various grains so that the animals are provided with minimum nutritional requirements which constitute the constraints. Various grains or foodstuffs have different prices and a pound of each grain contains different amounts of various nutrients—vitamins, minerals, calories, protiens, etc. Thus *the problem is to find a least-cost combination of various grains which meet the minimum nutritional requirements.*

Suppose a farmer feeds two types of grains, *A* and *B* to the animals reared by him. Constraint facing him is that the animals must be fed with certain minimum amounts of *three* nutrients, N_1, N_2 and N_3. Thus in this simplified example there are two variables, each with a price, represented by gains, three constraints regarding the minimum quantities of three nutrients which must be provided. The objective function is the minimization of cost of providing the required diet to the animals. What are the feasible solutions in this case and what will be the optimum solution is illustrated in Fig. 17.8 where the *X*-axis measures the quantity of grain *A* and the *Y*-axis measures the quantity of grain *B*. The line *EF* shows the various combinations of grains *A* and *B* which meet the minimum requirements of nutrient N_1. Similarly, the line *GH* represents the combinations of

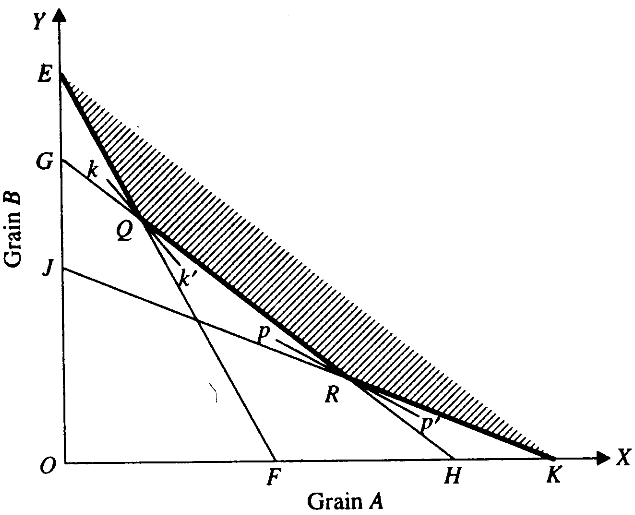


Fig. 17.8. *Solution of Diet Problem through Linear Programming*

two grains which provide the minimum requirements of nutrient N_2 and the line *JK* represents the combinations of the two grains which fulfil the minimum requirements of nutrient N_3. It should be noted that the steeper slope of the line *EF* shows that one pound of grain *A* contains relatively more amount of nutrient N_1 than grain *B*. Now, we are interested in obtaining the region of feasible solutions. Since any combination of grains lying below the lines showing three constraints will not meet the minimum nutritional requirements, the outer segments of the three lines which have been thickened constitute the boundary, the *combinations lying on it and to the right of it* will be feasible solutions. The thick line *EQRK* consisting of linear segments and stripped area to the right of it will form the region of feasible solutions and therefore any combination in it will ensure that when it is given to the animal it will provide the minimum amounts of nutrients required. But the farmer who is rearing the animals for the sake of making profits would like to minimise cost of providing the minimum required diet.

In order to find out the optimum or cost-minimizing combination of grains we need to bring in prices of the two grains. Let the prices of two grains be such that the iso-cost line has the slope indicated by the line *pp′* (various iso-cost lines with this slope can be drawn). In order to minimize costs, the farmer would try to reach the lowest possible iso-cost line. It will be noticed from the Fig. 17.8 that the boundary *EQRK* consisting of linear segments is hitting the iso-cost line *pp′* at point *R*. Thus, given the grain prices as indicated by the iso-cost line *pp′*, combination *R* occupies the lowest position in the zone of feasible solutions. Therefore, combination of grains represented by point *R* is optimum when the prices of grains are given by the slope of the iso-cost line *pp′*. It will be seen from the slope of the iso-cost line *pp′* that the price of grain *A* is relatively lower. Therefore, in the optimal solution *R*, the greater quantity of grain *A* is being fed. Now suppose that prices of grains are indicated by the iso-cost line *kk′*, according to which the price of grain *B* is relatively lower. With iso-cost line *kk′*, the optimum or cost-minimization of two grains will be *Q* where relatively greater quantity of grain *B* is being given.

An important point to note is that with iso-cost line *pp′* and optimum solution *R*, if the prices of grains change somewhat so that the iso-cost line changes a little but still hits the corner *R*, the optimum solution will remain unchanged. In other words, the prices can change somewhat without changing the optimum solution. However, if there is a drastic change in prices as when the iso-cost line switches from *pp′* to *kk′*, the optimum solution will change. It should be further noted that if the prices of two grains are such that the slope of the iso-cost line is identical with the line segment *QR* of the thick boundary line, then there will be *no single* optimum solution of the diet problem; either *R* or *Q* or any other point between them on the line segment *QR* will be the optimum.

PART IV

MARKET STRUCTURES AND PRICING OF PRODUCTS

- Market Structures and Concepts of Revenue
- Objectives of Business Firms
- Equilibrium of the Firm under Perfect Competition
- Pricing in a Perfectively Competitive Market
- Price and Output under Monopoly
- Price Discrimination
- Monopolistic Competition
- Price and Output under Oligopoly
- Pricinig in Practice : Mark-up Pricing and Sales Maximisation Model
- Administered Pricing : Price Regulation and Control

18

Market Structures and Concepts of Revenue

The determination of prices and outputs of various products depends upon the type of market structure in which they are produced, sold and purchased. In this connection economists have classified the various markets prevailing in a capitalist economy into (*a*) perfect competition or pure competition, (*b*) monopolistic competition, (*c*) oligopoly and (*d*) monopoly. Three market forms, monopolistic competition, oligopoly and monopoly, are generally grouped under the general heading of imperfect competition, since these three forms of market differ with respect to the degrees of imperfection in the competition in the market. Monopolistic competition is highly imperfect and monopoly is the most imperfect form of market structure.

But before explaining the salient features of various market structures, it will be useful to explain what is meant by market in economics.

Meaning of Market

Market is generally understood to mean a particular place or locality where goods are sold and purchased. However, in economics, by the term market we do not mean any particular place or locality in which goods are bought and sold. The idea of a particular locality or geographical place is not necessary to the concept of the market. What is required for the market to exist is the contact between the sellers and buyers so that transaction (*i.e.*, sale and purchase of a commodity) at an agreed price can take place between them. The buyers and sellers may be spread over a whole town, region or a country but if they are in close communication with each other either through personal contact, exchange of letters, telegrams, telephones, etc., so that they can sell and buy a good on an agreed price, the market would be said to exist. Further it is noteworthy that because in a market, there is close and free communication between various buyers and sellers price of a homogeneous commodity settled between different sellers and buyers tends to be the same. Thus, in the words of Cournot, a French economist, "*Economists understand by the term market not any particular market place in which things are bought and sold but the whole of any region in which buyers and sellers are in such free intercourse with one another that the price of the same goods tends to equality easily and quickly*".

Thus the essentials of a market are

(*a*) commodity which is dealt with;

(*b*) the esistence of buyers and sellers;

(*c*) a place, be it a certain region, a country or the entire world; and

(*d*) such contact between buyers and sellers that only one price should prevail for the same commodity at the same time.

CLASSIFICATION OF MARKET STRUCTURES

The popular basis of classifying market structures rests on two crucial elements, (1) the number of firms producing a product and (2) the nature of product produced by the firms, that is, whether it is homogeneous or differentiated. (3) the ease with which new firms can enter the industry. The price elasticity of demand for a firm's product depends upon the number of competitive firms producing the same or similar product as well as on the degree of substitution which is possible between the product of a firm and other products produced by rival firms. Therefore, a distinguishing feature of different market structures is the degree of price elasticity of demand faced by an individual firm.

We present in the table given below the classification of market structures based on the number of firms, the nature of product produced by them and whether entry into an industry is free or restricted.

Table 18.1. A Classification of Market Structures

Form of Market Structure	*Number of Firms*	*Nature of Product*	*Ease of Entry into an Industry*	*Price Elasticity of Demand for a product of an Individual Firm*	*Degree of Control over Price*
(*a*) Perfect Competition	A large number of firms	Homogeneous Product	Free Entry and Exit	Infinite	None
(*b*) Imperfect Competition					
(*i*) Monopolistic Competition	A large number of firms	Differentiated Products (which are close substitutes of each other)	Free entry in the sense that new firms can produce only close substitute	Large	Some
(*ii*) Pure Oligopoly (*i.e.*, Oligopoly without Product Differentiation)	Few firms	Homogeneous Product	Barriers to Entry	Small	Some
(*iii*) Differentiated Oligopoly (*i.e.*, Oligopoly with Product Differentiation)	Few firms	Differentiated Products (which are close substitutes of each other)	—do—	Small	Large
(*c*) Monopoly	One	Unique Product without Close Substitutes	Strong Barriers to Entry	Very small	Very large

The different market conditions prevail in various industries. In some of the market structures the forces of demand and supply do not work freely. In particular, the degree of competition faced by firms in various categories of market structure greatly differ. In fact, the market structure is determined by the degree of competition faced by firms in an industry. Traditionally, the market structure is classified into four categories: (1) Perfect Competition (2) Monopoly, (3) Monopolistic Competition, and (4) Oligopoly. Now a little explanation will be given at each form of market structure.

Perfect Competition

Perfect Competition is an extreme form of market structure in which forces of demand and supply work quite freely to determine the allocation of resources among different goods and distribution of income among factors. In this market structure, degree of competition is so high that no individual firm can influence the price of its product. There are a large number of firms producing an identical product. Each firm produces relatively a very small part of the total production of the product with the result that increases or decreases of output of the product by it makes hardly any difference to total supply of the product. This means no firm is able to affect the price of the product by varying its level of output. The firm working under market conditions of perfect competition, therefore, takes the price of the product as given and constant for it. It is therefore often referred to as *price taker*. The market condition that prevail under perfect competition are:

1. There are a large number of firms (sellers) and buyers and none of them can exercise any influence over the market price of the product which is determined by free working of demand for and supply of it.
2. Different firms produce and sell absolutely homogeneous products so that buyers have no preference for the product of any firm. There is no competitive advertising, no brands, no patents or any other feature that makes one firm's product look different from another.
3. There is free entry and free exit from the market or industry. That is, there are no barriers whatsoever to the entry of firms in the industry and to produce exactly the same product as produced by the existing firms. The firms, if they desire, can also easily leave the industry. Workers have perfect mobility, they can change employment from one firm or industry to another quite freely.
4. Lastly, both sellers and buyers, possess perfect knowledge and information about current prices in the industry and costs of production that have to be incurred on producing the product. In fact, all other economic facts are known to buyers and sellers.

The outcome of the above conditions of perfect competition is that a uniform price of the product prevails in the market. An individual firm can sell all it can produce and wants at the single uniform price. No producer or seller can sell any part of his output even at a slightly higher price. Any buyer can purchase any amount of the product it wants at the prevailing uniform market. He has not to pay even a slightly higher price than the current uniform price of the product.

It is important to understand the nature of competition prevailing in this market structure of perfect competition. The competition prevailing in this market structure is not of the nature of competition as is generally understood in popular usage. As is generally understood, the competition means the different firms in an industry compete with each other intensively to sell their brands of a product through advertising campaign in the newspapers, on TV and radio, offering some concessions and facilities to buyers, cutting prices, making attractive packages of their products etc. However, under perfect competition, competition through such means is not present. As mentioned above, in the perfectly competitive market structure there is no advertising, no brands, no trade names, and further there prevails a uniform price for the homogeneous product. Under perfect competition, since there is a very large number of firms, each producing no more than a fraction of total output of a product, no individual firm can influence the price of the product through any change in its output. Therefore, the firm recognise their inability to change the price and accept the prevailing market price of the product. As a result, neither sellers, nor buyers think about business or personal rivalry among themselves. Perfect competition is the most widely used model of economic behaviour and market structure. Of cource, it is an abstraction from reality but several economic predictions based on it have been found to be correct and reliable.

Monopoly

Monopoly is the other extreme form of market structure. In monopoly, there is a single producer and seller of a particular product which has no close substitutes. The monopoly has originated from the Greek words *Mono* meaning 'single' and *poly* meaning 'seller'. Thus, monopoly means a single seller. However, in economics absolute monopoly exists when the single seller sells a product which has not to face any competition from any close substitutes. Therefore, the monopolist does not have any rivals or competitors. This implies that the degree of competition in monopolistic market structure is nil or extremely small.

For a monopoly to get established, there must be strong barriers to the entry of firms that prevent other firms from entering the industry. If other firms can easily enter an industry, monopoly in the market cannot be sustained. It is important to understand that though under monopoly there are no direct rivals or competitors who sell the same or similar product, a monopolist has to face indirect competition. *In fact all goods and services compete with each other for a place in consumer's budget; some goods serve as close substitutes but none of them are perfect substitutes.*

In the real world, public utilities such as Delhi Vidyut Board supplying electricity to the city of Delhi, Mahanagar Telephone Nigam Limited (MTNL) providing telephone services to the residents of Delhi are some examples of monopoly, though they are regulated by the Government. Until recently (before the private buses were allowed to operate) Delhi Transport Corporation (DTC) had also a monopoly of bus transport in Delhi. Apart from these public utilities, which are regulated by the Government, very few pure monopolies exist in the real world.

Imperfect Competition

Imperfect competition is an important market category wherein individual firms exercise control over the price to a smaller or larger degree depending upon the degree of imperfection present in a case. Control over price of a product by a firm and therefore the existence of imperfect competition can be caused either by the 'fewness' of the firms or by the product differentiation. Therefore, imperfect competition form of market structure has several sub-categories. The following are three types of imperfect competition.

(*a*) Monopolistic Competition

(*b*) Pure oligopoly or Oligopoly without Product Differentiation

(*c*) Oligopoly with Product Differentiation

We explain below these three types of imperfect market structure.

(*a*) Monopolistic Competition

Between the two extreme market structures explained above, there is an important market structure which is known as monopolistic competition. Monopolistic competition in markets exists in which a large number of firms produce and sell products that are differentiated but close substitutes of each other. In this form of market structure there is a blending of elements of monopoly and competition. Monopoly exists in the sense that each of a large number of firms exercises sole control over the supply of its brand of the product. But, along with it, a firm in this market structure also faces competition form the other brands or varieties of the product which are close substitutes of its brand. In a way in the monopolistic competition, monopolies are competiting with each other.

Let us give some examples of monopolistic competition. There are more than 20 brands of bathing soap such as Lux, Hamam, Godrej, Palmolive, Dove, Jai, O.K. available in the Indian market. A firm called Hindustan Lever is the manufacturer of Lux and because it has patent rights over the trade name Lux, no one else can produce bathing soap named Lux. Therefore, Hindustan Lever has monopoly in the production of Lux. But it cannot be said to have absolute or pure monopoly because it has to compete with other varieties of bathing soap such as Hamam, Godrej, Palmolive available in the market and which are its close substitutes. Thus, products under monopolistic competition are differentiated but similar and close substitutes. The degree of competition is therefore quite high. Therefore, under monopolistic competition, prices of the products of different firms can be only slightly different from each other depending upon the goodwill or popularity they enjoy among consumers. Market structure of monopolistic competition prevails in the real word in several industries as those producing tooth pastes, tooth brushes, televisions, wooden furniture, electric fans, bulbs, cigarettes, etc.

(b) Oligopoly

Finally, we have a market structure in which there are a few firms supplying the product. The term "Oligopoly" is also derived from Greek. It comes from *Oligos* which means 'few' and *polein* which means 'selling'.

In pure oligopoly a few firms produce and sell homogeneous products. In India, in industries such as steel, cement, fertilizes there are not only few firms in them but their products are also almost identical. On the contrary, in differentiated oligopoly, a few firms produce product which are differentiated but close substitutes of each other.

Oligopoly with or without differentiation widely prevails in several industries in the Indian economy. A small degree of competition exists in this type of market structure. The most important features of oligopoly is that there is mutual interdependence among the firms in it. Firms in oligopoly are usually of a large size and each recognizes that its decisions will affect the other firms and therefore they will react to the moves made by it in a way that would in turn greatly affect it. An interesting example is the cold-drink industry in India, Pepsi-Cola, Campa Cola and Coca Cola. Coca Cola firm realises that any lower price fixed for its drink would affect Pepsi Cola and Campa Cola and therefore they would react to its pricing decision. There may be price war between them for some time but ultimately they would realise their interdependence and reach an understanding to charge a uniform price. But because of the mutual interdependence they would constantly watch others moves and react. Non-price competition such as advertising campaigns for their drinks to gain greater share of the market continues with great intensity.

CONCEPTS OF REVENUE

In the previous few chapters we discussed the nature of demand from the viewpoint of the consumer. But a producer or seller of a commodity is also very much concerned with the demand for it, mainly because revenue obtained by him from selling a good depends mainly upon the demand for the good. He is, therefore, interested in knowing what sort of demand curve faces him. The demand curve of the consumers for a product is the average revenue curve from the standpoint of the sellers, since the price paid by the consumers is revenue of the sellers. Besides average revenue, there are other concepts of revenue, namely, total revenue and marginal revenue. We shall explain below these concepts of revenue and their relationship with each other and with the elasticity of demand.

Average Revenue

Price paid by the consumer for a product forms the revenue or income of the seller. The total sales value received by the seller from selling a given amount of the product is called *total revenue.* If a seller sells 15 units of a product at price of Rs. 10 per unit, his total revenue from this sale is Rs. 150. On the other hand, average revenue is revenue earned *per unit of output.* Average revenue can be obtained by dividing the total revenue by the number of units produced and sold. Thus,

$$\text{Average revenue} = \frac{\text{Total revenue}}{\text{Total output sold}}$$

$$AR = \frac{TR}{Q}$$

where AR stands for average revenue, TR for total revenue and Q for total output produced and sold.

In our above example, when total revenue of Rs. 150 is received from selling 15 units of the product, the average revenue will be equal to Rs. 150/15 = Rs. 10. Rs. 10 is here the revenue earned per unit of output. Now, the question is whether average revenue is different from price or these two concepts mean the same thing. *If a seller sells various units of a product at the same price, then average revenue would be the same thing as price.* But when he sells different units of a given product at different prices, then the average revenue will not be equal to price. An example will clarify this point. Suppose a seller sells 2 units of a product, both at a price of Rs. 10 per unit. Total revenue of the seller will be Rs. 20 and the average revenue will be 20/2 = Rs. 10. Thus, average revenue is here equal to the price of the product. Now suppose that the seller sells two units of his product, one unit to the consumer *A* at price Rs. 12 and one unit to the consumer *B* at price Rs. 10. His total revenue from the sale of two units of the product will be Rs. 22. Average will be here equal to 22/2 = Rs. 11. Thus, in this case when two units of the product are sold at different prices, average revenue is not equal to the prices charged for the product.

But in the actual life we find that different units of a product are sold by the seller at the same price in the market (except when he discriminates and charges different prices for different units of good). Average revenue therefore equals price. Thus, in economics we use average revenue and price as synonyms except when we are discussing price discrimination by the seller. Since the buyer's demand curve represents graphically the quantities demanded or purchased by the buyers at various prices of the good, it also, therefore, shows the average revenue obtained from producing and selling various amounts of the good. This is because the price paid by the buyer is revenue from seller's point of view. Hence, *average revenue curve of the firm is really the same thing as the demand curve of the consumers.*

Marginal Revenue

On the other hand, marginal revenue is the *net revenue* earned by selling an *additional unit* of the product. In other words, *marginal revenue is the net addition made to the total revenue by selling one more unit of a commodity.* Putting it in algebraic expression, marginal revenue is the addition made to the total revenue by selling n units of a product instead of $n-1$ where n is any given number. If a producer sells 10 units of a product at price Rs. 15 per unit, he will get Rs. 150 as the total revenue. If he now increases his sales of the product by one unit and sells 11 units, suppose the price falls to Rs. 14 per unit. He will, therefore, obtain total revenue of Rs. 154 from the sale of 11 units of the good. This means that 11th unit of output has added Rs. 4 to the total revenue. Hence Rs. 4 is here the marginal revenue.

Total revenue when 10 units of a commodity are sold at price of Rs. 15

$$= 10 \times 15 = \text{Rs. } 150$$

Total revenue when 11 units of a commodity are sold at price of Rs. 14

$$= 11 \times 14 = \text{Rs. } 154$$

$$\text{Marginal revenue} = 154 - 150 = \text{Rs. } 4$$

The word *net* in the first definition of marginal revenue given above is worth noting. The full understanding of the word 'net' in the definition will reveal why the *marginal revenue is not equal to the price.* The question is, taking our above numerical example, why the marginal revenue of the 11th unit is not equal to the price of Rs. 14 at which the 11th unit is sold. The answer is that the 10 units which were sold at the price of Rs. 15 before will now *all* have to be sold at the reduced price of Rs. 14 per unit. This will mean the loss of one rupee on *each* of the previous 10 units and, therefore, total loss on previous 10 units due to price fall of Re 1 per unit will be equal to Rs. 10. The loss in revenue incurred on the previous units is due to the fact that the selling of the additional 11th unit reduces the price from Rs. 15 to Rs. 14 for all. Thus in order to find out net addition made to the total revenue by the 11th unit, the loss of revenue (Rs. 10) on previous units should be deducted from the price of Rs. 14 at which the 11th unit is sold along with others. The marginal revenue in this case is, therefore, equal to Rs. 14 − 10 = 4. Marginal revenue is thus less than the price at which the additional unit is sold.

It is clear from above that marginal revenue can either be found directly by taking out the difference between total revenue before and after selling the additional unit, or it can be obtained by subtracting the loss in revenue on previous units due to the fall in price from the price at which the additional unit is sold.

Therefore,

Marginal revenue = difference in total revenue by increasing production and sales from $n - 1$ units to n units.

= price of the additional unit minus loss in revenue on previous units resulting from the price reduction.

It follows from above that when the price falls as additional unit is sold, marginal revenue is less than price. But when the price remains the same as additional unit is sold, as under perfect competition, the marginal revenue will be equal to price or average revenue, since in this case there is no loss incurred on the previous units due to the fall in price. *The relationship between average revenue and marginal revenue is the same as between any other average and marginal values. When average revenue falls, marginal revenue is less than the average revenue. When average revenue remains the same, marginal revenue is equal to average revenue.*

If TR stands for total revenue and Q for output, the marginal revenue (MR) can be expressed as follows:

$$MR = \frac{\Delta TR}{\Delta Q}$$

$\frac{\Delta TR}{\Delta Q}$ indicates the slope of total revenue curve. Thus if the total revenue curve is given to us, we can find out marginal revenue at various levels of output by measuring the slope at the corresponding points on the total revenue curve.

Average Revenue and Marginal Revenue Under Monopoly and Monopolistic Competition

The concepts of total, average and marginal revenues under imperfect competition (that is, monopoly and monopolistic competition) are explained in tabular form below in Table 18.2. It will be seen from this table that as more units of a commodity are produced and sold, price of the commodity falls. This is because when monopoly, monopolistic competition or oligopoly prevails in the market and more ourput of a commodity is produced and sold by a firm, its price (or AR) falls. We have seen above that monopoly, monopolistic competition and oligopoly are different forms of imperfect competition, monopoly being the extreme case.

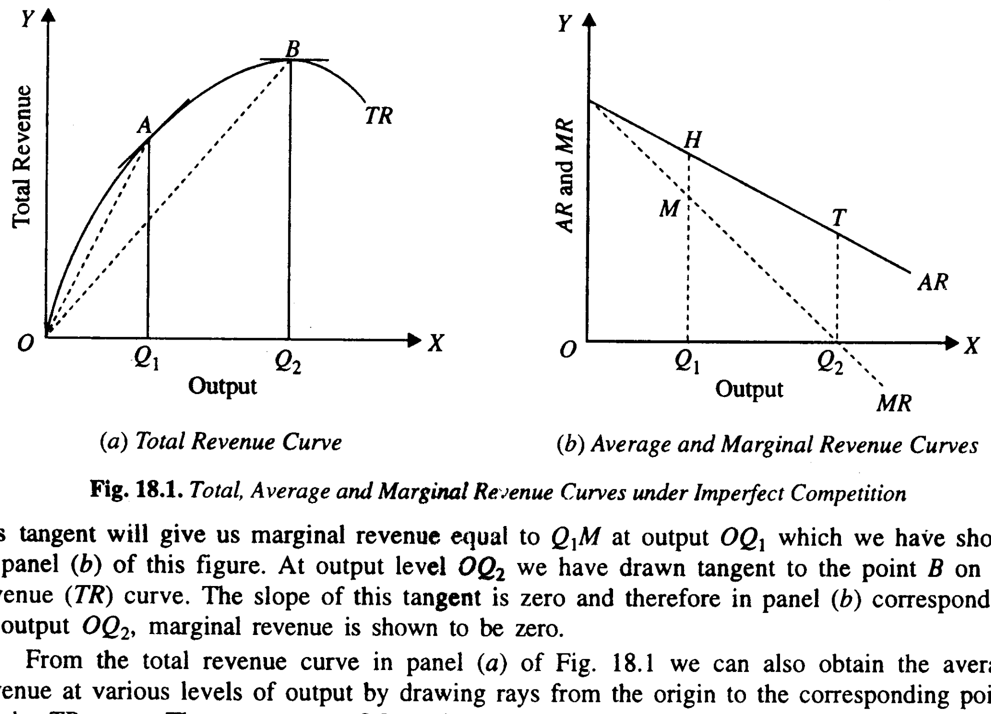
Table 18.2: Total, Average and Marginal Revenues Under Imperfect Competition

No. of Units Sold	*Total Revenue*	*Average Revenue or Price*	*Marginal Revenue (Addition made to Total Revenue) (in Rs.)*
I	*II*	*III*	*IV*
1	16	16	16
2	30	15	14
3	42	14	12
4	52	13	10
5	60	12	8
6	66	11	6
7	70	10	4
8	72	9	2
9	72	8	0
10	70	7	–2

It will be seen in the Column III of the table that price (or average revenue) is falling as additional units of the product are sold. Marginal revenue can be found out by taking out the difference between the two successive total revenues. Thus, when 1 unit is sold, total revenue is Rs. 16. When 2 units are sold, price (AR) falls to Rs. 15 and total revenue increases to Rs. 30. Marginal revenue is therefore here equal to $30 - 16 = 14$, which is recorded in Column IV. When 3 units of the product are sold, price falls to Rs. 14 and total revenue increases to

Rs. 42 and hence marginal revenue is now equal to Rs. (42 – 30) = Rs. 12 which is again recorded in Col. IV. Likewise, marginal revenue of further units can be obtained by taking out the difference between two successive total revenues. Marginal revenue becomes negative when total revenue declines. Thus, when in our Table 18.1 quantity sold is increased from 9 units to 10 units, the total revenue declines from Rs. 72 to 70 and therefore the marginal revenue is negative and is equal to –2.

In all forms of imperfect competition, that is, monopolistic competition, oligopoly and monopoly, average revenue curve facing an individual firm slopes downward. This is because under imperfect competition when a firm increases its level of output, the price of its product falls. The case, when average revenue (or price) falls as additional units of the product are sold in market has been graphically represented in Fig. 18.1. In panel (*a*) of Fig. 18.1 the total revenue curve, (*TR*) is rising but at a declining rate. This means marginal revenue, which is equal to the slope of the total revenue curve $\left(MR = \frac{\Delta TR}{\Delta Q}\right)$, *will be declining throughout.* Thus, corresponding to output OQ_1, we have drawn a tangent to point *A* on the total revenue curve *TR*. Measuring the slope of



(*a*) *Total Revenue Curve* (*b*) *Average and Marginal Revenue Curves*

Fig. 18.1. *Total, Average and Marginal Revenue Curves under Imperfect Competition*

this tangent will give us marginal revenue equal to Q_1M at output OQ_1 which we have shown in panel (*b*) of this figure. At output level OQ_2 we have drawn tangent to the point *B* on the revenue (*TR*) curve. The slope of this tangent is zero and therefore in panel (*b*) corresponding to output OQ_2, marginal revenue is shown to be zero.

From the total revenue curve in panel (*a*) of Fig. 18.1 we can also obtain the average revenue at various levels of output by drawing rays from the origin to the corresponding points on the *TR* curve. Thus, at outputs OQ_1 and OQ_2 we have drawn the rays *OA* and *OB* from the origin to the *TR* curve. Measuring the sopes of these rays *OA* and *OB* gives us average revenue Q_1H at output Q_1 and Q_2T at output level OQ_2 in panel (*b*) of Fig. 18.1. By drawing rays from the origin to various points on the *TR* curve, it will be found that the slopes of the ray to various points will be declining as output sold increases. In panel (*b*) of Fig. 18.1, it will be observed that average revenue curve (*AR*) is falling downward and marginal revenue curve (*MR*) lies below it.

The fact that *MR* curve is lying below *AR* curve indicates that marginal revenue declines more rapidly than average revenue. When OQ_2 units of the good are sold, marginal revenue is zero. If the quantity sold is increased beyond OQ_2, marginal revenue becomes negative.

Average Revenue and Marginal Revenue Under Perfect Competition

When there prevails perfect competition in the market for a product, demand curve facing an